A Synopsis of research work

for

PhD Degree

by

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(Registration No. : R-14RS09140064 dated August 12, 2016)

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October, 2018

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(within 5000 words)

Title: **Nonequilibrium Phase Transition in Ferromagnetic Model Systems.**

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Degree for which submitted: **Doctor of Philosophy**

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1. Introduction:

To explain the physical properties and behaviour of various physical systems (in bulk), on the basis of the dynamical behaviour of the microscopic constituents Statistical Mechanics plays an important role. The scope of its formalism covers almost every natural phenomenon whether in equilibrium or in nonequilibrium and in any state or phase of matter whatsoever. Thermodynamic behaviours of various physical systems in equilibrium conditions had long been studied and understood using the well established principles of Equilibrium Statistical Mechanics[23]. Though these principles of equilibrium statistical mechanics are extremely powerful, still theoretically only few systems and their behaviours, under ideal conditions, can be described by these principles. Major difficulties arise when it becomes necessary to deal with a large number of microscopic components and under varied conditions or situations. Again more often we come across situations where systems are kept away from equilibrium by externally applied time dependent excitations such as electric or magnetic fields which also tend to keep the thermodynamic variables describing the system, time dependent. These facts relate the system more close to the real situations where the dynamical behaviours of such systems play the major roles. With the growing demand of modern technological advances, necessity of understanding of dynamical behaviours of various systems under varied conditions has risen up. Thus a thrust of attention of researchers has been observed in recent three or four decades in the field of nonequilibrium phenomena. There are two major branches of studies in this field: one deals with experiments while the other deals with theoretical results and simulations using computations. With the advent of modern high speed computers, simulations have become easier and using different kinds of theoretical models dynamical behaviour of various kinds of systems are being studied in details. Among various kinds of nonequilibrium systems, ferromagnetic systems are quite important for many reasons [1, 2, 3, 4, 5, 6, 7, 8, 14, 15, 16, 17, 18, 19]. Investigation of nonequilibrium behaviour of a ferromagnet is extremely useful in present era of modern technological advances. In the field of spintronics and magnonics[9, 10, 11, 12, 13] spin waves are excited directly in materials having intrinsic spins, which have a great technological implication now-a-day. Also the visible, (statistically) orderly outcomes of self-organization and the common principles behind similar patterns in nature have attracted the researchers' attention into the field of studying pattern formation and its evolution. This is where and why I focus my attention for research.

Different kinds of nonequilibrium phases and their dynamical behaviours have been studied in ferromagnetic systems by varying the values of different parameters that specify the dynamical states. It is observed that a ferromagnetic system undergoes nonequilibrium phase transition depending on the values of field amplitudes of externally applied magnetic field, temperature, anisotropy etc.

In this synopsis of my thesis, I have mainly reported the results of the nonequilibrium phase transition, the dynamical symmetry breaking observed in discrete spin models (Ising, Blume-Capel etc.) driven by the propagating and standing magnetic field wave. The technique employed here is Monte-Carlo simulation using Metropolis Single spin flip algorithm.

Before going to my research details I am giving a brief introduction to Hamiltonian of a system, system in nonequilibrium, forms of propagating and standing waves and Monte-Carlo methods. The Hamiltonian of a system describes the total energy (kinetic + potential) of the system. In a magnetic system total energy consists of mainly two parts: (a) spin-spin interaction energy and (b) spin-magnetic field interaction energy.

The equilibrium or the steady state condition of a system is defined as the state of the system where the total energy do not vary with time under certain prevailing conditions. A system is generally in nonequilibrium when there exists any time dependent driving force which tends to keep the system away from equilibrium. Generally a flow of energy or mass etc. through the system is associated with the system in nonequilibrium. The waves are the disturbances in a medium or system. The forms of 1-Dimensional propagating and standing waves are respectively as follows:

$$f(x,t) = Acos(2\pi\nu t - 2\pi x/\lambda),$$

$$f(x,t) = Asin(2\pi\nu t)cos(2\pi x/\lambda),$$

where f(x,t) represents the instantaneous value of the disturbance at position x, A is the amplitude of the disturbance, ν is the frequency of the wave and λ is the wavelength of the wave. Monte-Carlo simulation is a widely used technique, based on computer algorithms, to generate dynamics, in a system, governed by any dynamical equation. This mainly rely on repeated random sampling to obtain numerical results[21]. In principle, Monte Carlo methods can be used to solve any problem having a probabilistic interpretation.

2. Methodology and computational details:

I have studied the nonequilibrium phase transitions in model ferromagnets driven by externally applied propagating or standing magnetic wave and uniformly oscillating magnetic field. I obtained the results using computer simulation. I have chosen the simplest model, the 2-Dimensional Ising model[23], to study the nonequilibrium ferromagnetic behaviour under external stimulations. In an Ising type ferromagnet in two dimensions spins are considered to be arranged in a square lattice (there may be different kinds of lattice structures also such as triangular or honey comb lattice). These spins interact with each other giving rise to the spin-spin interaction energy and also interact with the externally applied magnetic fields giving rise to the spin-field interaction energy. Thus, there are mainly two energy terms in the Hamiltonian of the model systems. Also there may be additional energy term which arises when there is anisotropy in spins as in case of Blume-Capel ferromagnet.

The basic methodology for this study is to simulate the systems using proper algorithm for dynamic evolution. Monte-Carlo method is the most widely used technique to simulate such dynamics (which can incorporate individual spin fluctuation). In this method the probability of a single spin flip, called the Metropolis rate, is calculated at a certain temperature T at any time t for spins at every lattice site. The Metropolis rate is given by the equation [21],

$$W(s_i \rightarrow s_f) = Min[exp(-\frac{\Delta E}{k_B T}), 1]. \quad (1)$$

where, ΔE is the change in energy due to spin flip from initial state i to final state f, k_B is the Boltzmann constant and T is the thermodynamic temperature of the system. Min represents the minimum value of the two. The spins are then updated according to the Metropolis rate and a new configuration of spin state is obtained from initial configuration, thus dynamics of spin evolution is obtained. Various dynamical quantities are then calculated from the final spin configuration when the steady state is achieved. The unit of time is measured in $MCSS$[21], which is defined as the time in which total number of spins in the system is updated once.

In this study of nonequilibrium phase transition in Ising ferromagnet I defined an appropriate *Order parameter* for phase transition. To show the transition I have also calculated different quantities such as Variance, the dynamic specific heat etc. The variation of these parameters with temperature are plotted to find the dynamic transition temperature for the phase transition. Also lattice morphologies of spins at different values of temperatures and different parameters and dynamic symmetry breaking are studied in our different studies.

3. The aim of the thesis and its achievements in that perspective: Summary of research work:

3.1. Effect of Standing magnetic wave on Ising ferromagnet: Nonequilibrium phase transition:

Here I have modelled and studied the dynamical response of an Ising ferromagnet to a plane polarised standing magnetic field wave by *Monte Carlo simulation* in two dimensions. It is, here, worthy to mention that similar studies in Ising ferromagnet have been done with propagating magnetic wave[20] and different dynamical phases have been observed. The Hamiltonian (*time dependent*) of a two dimensional Ising ferromagnet, having uniform nearest neighbour spin-spin interaction in presence of an external standing magnetic wave is represented by,

$$H(t) = -J\Sigma\Sigma' s^z(x,y,t)s^z(x',y',t) - \Sigma h^z(x,y,t)s^z(x,y,t). \tag{2}$$

where $s^z(x,y,t)$ is the Ising *spin variable* (± 1) at lattice site (x,y) at time t. The summation Σ' extends over the nearest neighbour sites (x',y') of given site (x,y). $J(>0)$ is the *ferromagnetic Spin-Spin interaction strength* between the nearest neighbours. It is considered to be uniform over the whole lattice for simplicity. $h^z(x,y,t)$ is the *magnetic field* at site (x,y) at time t, which has the following form of Standing wave,

$$h^z(x,y,t) = h_0 sin(2\pi ft)cos(2\pi x/\lambda) \tag{3}$$

The h_0, f and λ represent respectively *the field amplitude, the frequency and the wavelength* of the standing magnetic wave. The standing magnetic wave is extended along the x direction. Our aim is to know how the Ising ferromagnet, driven by standing magnetic wave, behaves at finite temperatures and how can I quantify the differences in dynamical patterns formed by standing magnetic wave with that formed by propagating magnetic wave? In this study a 100×100 square lattice is taken with *open boundary conditions*. Unlike in periodic boundary conditions, where translational invariance is kept along the lattice directions, this type of open boundaries are useful to study the effect of boundaries in the dynamics of driven system. Spin are updated *randomly* at any given site according to the Metropolis single spin flip algorithm given by the equation(1). In the steady state, I have detected two main dynamical phases namely, *oscillating spin clusters* and *pinned* (see figs.1a,1b). The pinned phase, with asymmetric and static arrangement of spins is the dynamically ordered phase having non-zero value of average magnetisation. The oscillating spin clusters phase consists of many parallel band shaped spin clusters and has zero average magnetisation. As the system is cooled from high temperature to low temperature, the dynamic order parameter becomes non-zero below a certain transition temperature (see fig.2a.), thus the system undergoes a phase transition from oscillating spin clusters phase to pinned phase. Here, the time averaged magnetisation, over a full cycle of magnetic field oscillations, play the role of the *dynamic order parameter*. The dynamic transition seems to be of continuous nature and the dynamic transition temperature (T_d) depends on the values of the amplitude (h_0) and the wavelength (λ) of the standing magnetic field wave at a single frequency. The transition is detected by studying (see figs.2b-2d) the temperature dependences of the variance of the dynamic order parameter, the derivative of the dynamic order parameter with respect to temperatue and the dynamic specific heat. These parameters show sharp peak or dip near transtion temperature. A comprehensive phase boundary (see fig.4 Right) is drawn for two different wavelengths of the magnetic wave in T_d versus h_0 plane. The phase boundary is

observed to shrink towards asymetric ordered phase for shorter wavelength. Unlike the propagating phase where instantaneous line magnetisation oscillates (see fig.3.) with the same amplitude at all lattice sites, the same oscillates coherently in loops with different amplitudes at different lattice sites along the standing wave. These loops are formed between two nodes (having almost zero value of instantaneous magnetisation at all times). It is detected that the spins flip more frequently at the nodes of the standing wave (see fig.4.Left). With open boundary condition applied to the lattice, the system achieved steady state after longer time as compared to that observed in the periodic boundaries applied to the lattice. Apart from these, the nature of transition is found similar to the earlier studies with periodic boundaries applied in the case of propagating wave[20].

This present study is a simple statistical mechanical approach of achieving various dynamical modes of Ising ferromagnet irradiated by a standing magnetic wave, just to have a preliminary notion about the behaviour of a ferromagnetic sample placed in intense optical pattern. It should be mentioned here that the earlier studies[6] , on the dynamic transition, in the Ising ferromagnet driven by oscillating (in time but uniform over space) magnetic field, reported the presence of discontinuos transition and located a tricritical point on the phase diagram. Later on, the studies[7] on the distribution of dynamic order parameter with much improved statistics showed the absence of any discontinuous transition. So, to identify any tricritical behaviour (or discontinuous transition, if any), one should study this with much improved statistics, which is beyond the scope of our computational facilities. Here, I do not make any such comment on the presence/absence of any tricritical point.

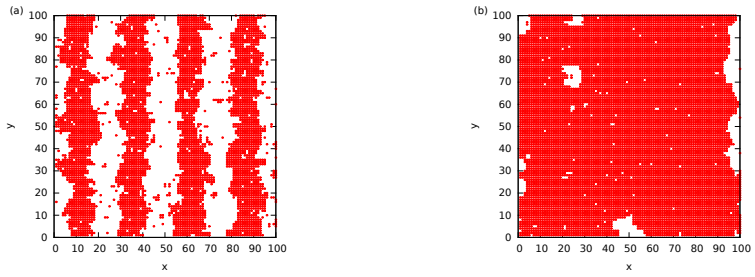

Figure 1: Dynamical lattice morphology for frequency $f = 0.01$ and field amplitude $h_0 = 0.6$. (a) Oscillating spin clusters phase (temp.T=2.0). (b) Pinned phase (temp. T=1.5). In these figures 'dots' denote upspin states.

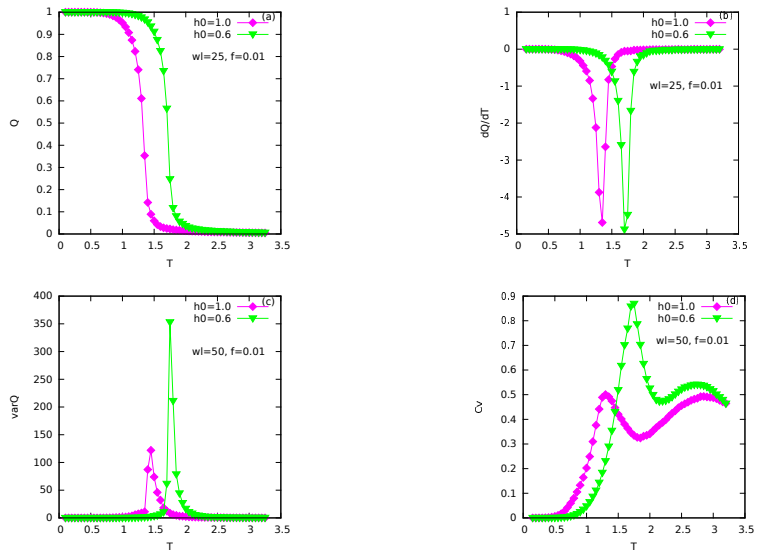

Figure 2: Temperature (T) variations of **(a)** Q, **(b)** $\frac{dQ}{dT}$, **(c)** $L^2\langle(\delta Q)^2\rangle$ and **(d)** C_v for two different values of standing magnetic field amplitude $h_0 = 0.6$ *and* 1.0. Here Q is the order parameter, L is the lattice size and C_v is the specific heat. The frequency is $f = 0.01\ MCSS^{-1}$. Figs.(a) and (b) are for $\lambda = 25$ while (c) and (d) are for $\lambda = 50$. In figs. 'wl' represents wavelength (λ).

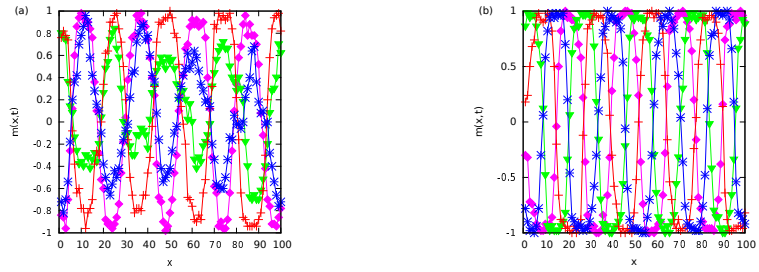

Figure 3: Periodic variation of *instantaneous line magnetisation* $m(x,t)$ at different lattice sites (x); (a) *for standing wave*, (b) *for propagating wave* in disordered phase, temperature $T = 2.00$ in units of J/k_B. Different symbols represent different times *magenta diamond* at 199900 MCSS, *green downtriangle* at 199925 MCSS, *red +* at 199950 MCSS, *blue asteriks* at 199975 MCSS, where time period of magnetic field oscillation is 100 MCSS. The standing wave is along x axis and the propagating wave propagates along x axis. Here the values field amplitude h_0, frequency f and wavelength λ are 0.6 J, 0.01 $MCSS^{-1}$& 25 *lattice units* respectively..

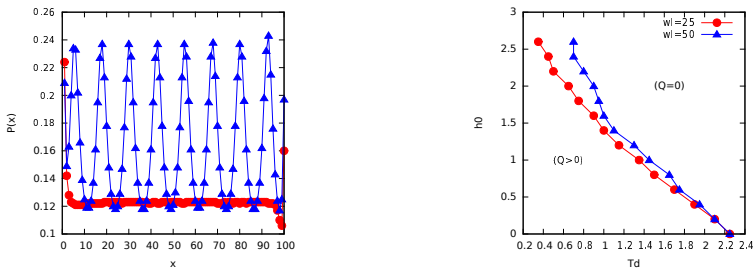

Figure 4: **(Left)**Probability $P_s(x)$ of spin flips at different lattice sites along x axis for standing magnetic wave (*blue triangle*) and for propagating magnetic wave (*Red bullet*) respectively in disordered phase, temperature $T = 2.00$ in units of J/k_B. The standing wave is along x axis and the propagating wave propagates along x axis. Here the values of field amplitude h_0, frequency f and wavelength λ are 0.6 J, 0.01 $MCSS^{-1}$& 25 *lattice units* respectively. **(Right)**Phase diagram (dynamic transition temperature T_d vs. field amplitude h_0) for two different wavelength (λ = 25 (*red dot*) & 50 (*blue triangle*)) of the standing magnetic field wave. The frequency of the standing wave is $f = 0.01$ $MCSS^{-1}$..

3.2. Dynamical modes in Blume-Capel ferromagnet driven by propagating and standing magnetic wave:

Ferromagnetic systems having single site anisotropy can be modelled and simulated using Blume-Capel model [24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39]. I have studied the dynamical response of the Blume-Capel ($S = 1$) ferromagnet in the presence of the plane propagating (with fixed frequency and wavelength) and standing magnetic field waves separately in two dimensions by Monte Carlo simulation. The energy or Hamiltonian of the two dimensional Blume-Capel ferromagnet in the presence of magnetic field is given by the equation:

$$E(t) = -J\Sigma\Sigma' s_z(x,y,t)s_z(x',y',t) + D\Sigma(s_z(x,y,t))^2 - \Sigma h(x,y,t)s_z(x,y,t) \qquad (4)$$

Here $s_z(x,y,t)$ represents the z component of the Spin ($S = 1$) at any position (x,y) at time t. $S_z(x,y,t)$ may take values from any of $-1,\ 0\ or\ +1$. The summation Σ' extends over the nearest neighbour sites (x',y') of given site (x,y). $J(> 0)$ is the uniform (over the space) ferromagnetic spin-spin interaction strength between the nearest neighbours. D represents the strength of anisotropy. $h(x,y,t)$ is the *magnetic field* at site (x,y) at time t, which has the following forms for two kinds of magnetic wave chosen here; e.g. propagating wave and standing wave respectively,

$$h(x,y,t) = Hcos(2\pi ft - 2\pi y/\lambda) \qquad (5)$$

$$h(x,y,t) = Hsin(2\pi ft)sin(2\pi y/\lambda) \qquad (6)$$

The H, f and λ represent respectively *the field amplitude, the frequency and the wavelength* of the magnetic field wave (both propagating and standing). Here I have chosen periodic boundary condition in both directions of a 100×100 square lattice to preserve the translation invariances in the system.

As in my previous study I have employed the Metropolis single spin flip algorithm with parallel spin updating rule is used. It is observed that the different dynamical states of the system depend on the values of the strength of anisotropy, temperature of the system and the amplitude of the magnetic field waves (both propagating and standing). Mainly two different dynamical phases have been identified (see fig.5.) in both cases. When the values of the strength of anisotropy and the amplitude of the propagating magnetic field are kept

fixed the system undergoes a dynamical phase transition from a driven propagating spin wave phase to a spin frozen or pinned phase as the system is cooled down. In the driven spin wave state, the alternate spin-bands of $S_z = +1$ and $S_z = -1$ move coherently along with the propagating wave. The time averaged magnetisation over a full cycle of the propagating magnetic field plays the role of the dynamic order parameter here. Thus, the spin frozen or pinned phase has nonzero value of the dynamical order parameter, whereas the dynamical order parameter becomes zero in the phase of coherent propagation of the spin-bands. As the system is cooled down from higher temperature this dynamical order parameter takes a nonzero value (see fig.7.) at the transition temperature. The variance of the order parameter gets sharply peaked (see fig.7.) at the transition point. The transition temperature is found from the peak position of the variance of the dynamic order parameter. It is observed that this dynamic transition temperature depends on the amplitude of the propagating magnetic field wave and the strength of the anisotropy. A comprehensive phase diagram (see fig.8.(Left)) is thus plotted in the plane formed by the amplitude of the propagating wave and the temperature of the system. It is found that the phase boundary shrinks inward (lower temperature) as the strength of the anisotropy increases. The phase boundary is also drawn in the plane described by the strength of anisotropy and temperature (see fig.8.(Right)). This phase boundary also shrink inward (low temperature and anisotropy) as the amplitude of the propagating magnetic field wave increases. Next, I studied the dynamical responses of the Blume-Capel model with the application of standing magnetic wave. Two distinct phases are also observed here. The high temperature phase (see fig.5.(Right)) is quite different from that observed in the case of propagating magnetic wave. The standing wave (non propagating) of alternate spin bands are formed in this case in contrast to the propagating wave as in the earlier studies. The dynamical phase boundaries are plotted in the $H - T$ and $D - T$ planes (see fig.10.). The qualitative behaviours are same as observed in the case of propagating magnetic wave. It may be noted that the sharp peak of the variance at the transition point is a signature of the divergence (in the thermodynamic limit) of the dynamic susceptibility. The Blume-Capel model shows tricritical behaviour in the equilibrium phase transition. However, in this particular nonequilibrium case, the transition seems to be continuous irrespective of the value of the strength of the anisotropy. A qualitative understanding of the existence of the dynamical phases may be as follows: at low temperature the values D and H are inadequate for spin flip. As a result one obtains the spin frozen phase. On the other hand if the temperature is high the same set of values of D and H become strong enough to flip the spins. In this phase spins are flexible enough to respond to the variation of propagating and standing magnetic wave. As a result, the propagating magnetic wave yields propagating spin bands and standing magnetic wave produces standing spin bands in the lattice.

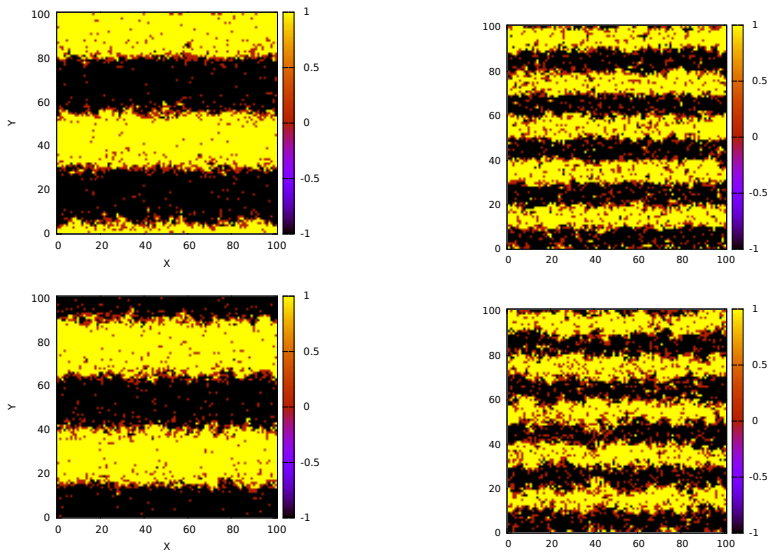

Figure 5: Coherent propagation of driven spin-wave shown here for two different values of time for *propagating wave*. Top-left is for t=2000 and bottom-left is for t=2070. Here, $D = 1.0$, $H = 1.5$, $T = 1.0$. Morphologies of *standing wave* dynamical modes (non-propagating) in disordered phase for two different times. Top-right one is at $t = 1000$ MCSS and the bottom-right one is taken at $t = 1070$ MCSS. Here, $H = 1.5$, T=1.5 and D=0.5 $\lambda = 20$.

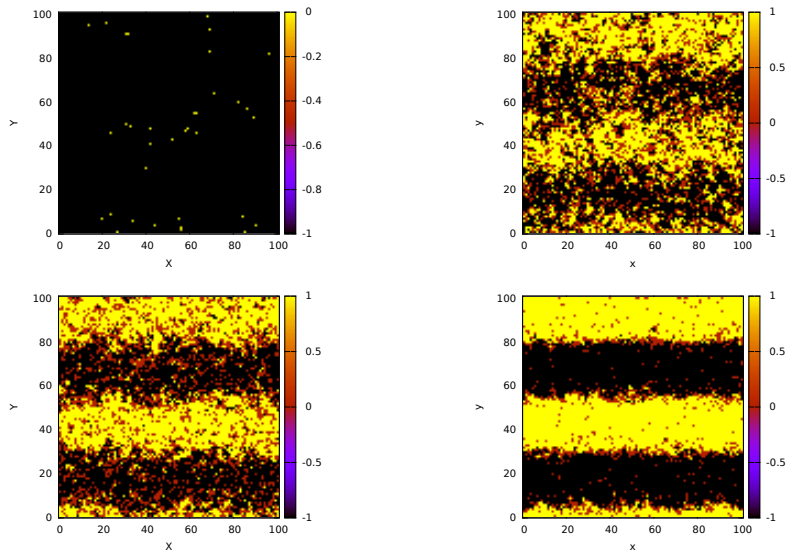

Figure 6: Morphology of the lattice (value of $S_z(x, y, t = 2000)$) for different values of T, H and D in case of propagating wave. Clockwise from top-left (i) D=1.0, H=0.5, T=0.5 (ii) D=0.1, H=0.3, T=1.9, (iii) D=1.0, H=1.5, T=1.0 and (iv) D=1.0, H=0.5, T=1.5

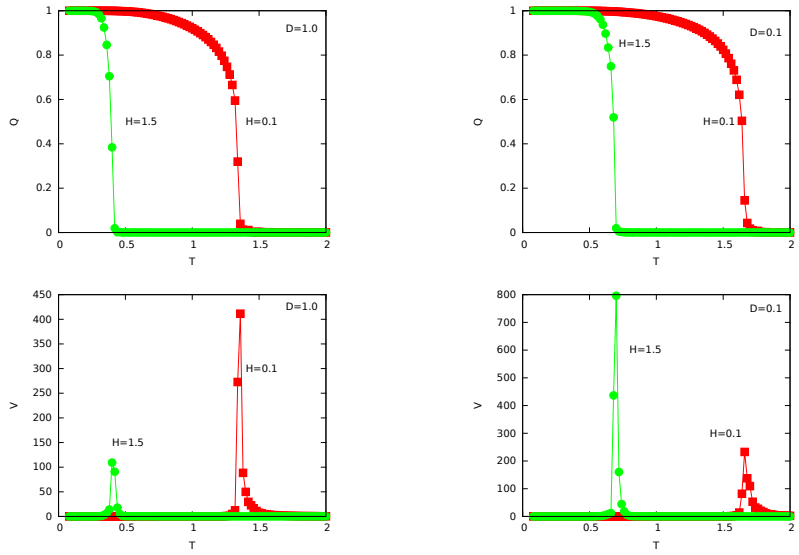

Figure 7: Temperature dependence of Q and V for two different values of H in case of propagating wave. Here, D=1.0 (left) and D=0.1 (right).

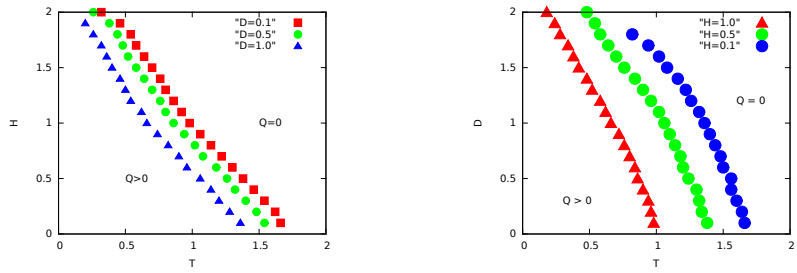

Figure 8: Phase diagram in the H-T plane and D-T plane in the case of propagating wave. Different symbols denote different values of D or H respectively (mentioned in the figure).

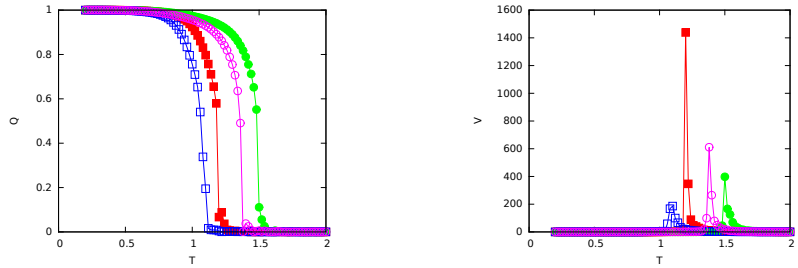

Figure 9: Q and V are plotted against temperature (T) for different values of field amplitude (H) and anisotropy (D) in the case of standing wave of wave length (λ) 20. Here, different symbols represent different values of H and D. (i) Red filled square (H=1.0, D=0.1) (ii) Blue open square (H=1.0, D=0.5), (iii) Green filled circle (H=0.5, D=0.1) and (iv) Open magenta circle (H=0.5, D=0.5)

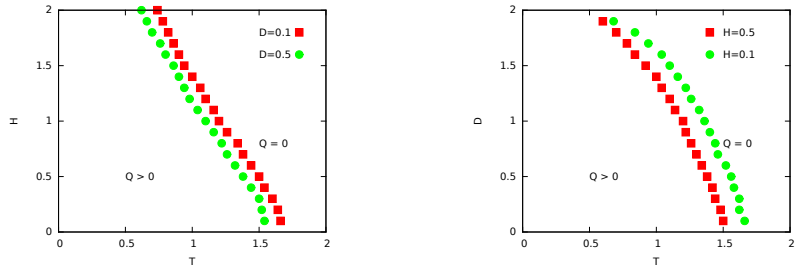

Figure 10: Phase diagram in the H-T and D-T plane in the case of standing wave. Different symbols denote different values of D or H respectively (mentioned in the figure). Here $\lambda = 20$.

3.3. Nonequilibrium phase transition in driven Spin-S Ising ferromagnet:

So far I have studied a specific type of ferromagnetic system under nonequilibrium condition; viz. spin-$\frac{1}{2}$ (Ising)[40] or spin-1 (Blume-Capel)[41] system. However, it is interesting to know the dynamic behaviour of a general spin-S ferromagnetic system having $(2S+1)$ number of spin states or degeneracies. *Is there any universal characteristic for dynamical phase transition under time dependent driving field for such system of general spin-S ferromagnet?* Our present study aims at this specific question. The dynamical response of S- spin Ising ferromagnet to the plane propagating wave, standing magnetic field wave and uniformly oscillating field with constant frequency are studied here separately in two dimensions by extensive Monte Carlo simulation with Metropolis algorithm. The *time dependent* Hamiltonian of a two dimensional Ising ferromagnet, having S numbers of spin states is represented by,

$$H(t) = -J\Sigma\Sigma' s^z(x,y,t)s^z(x',y',t) - \Sigma h^z(x,y,t)s^z(x,y,t). \tag{7}$$

Here $s^z(x,y,t)$ is the z component of the Ising *S-state unit spin variable* at lattice site (x,y) at time t. The summation Σ' extends over the nearest neighbour sites (x',y') of a given site (x,y). $J(>0)$ is the uniform *ferromagnetic spin-spin interaction strength* between the nearest neighbour pairs of spin. The externally applied *magnetic field*, $h^z(x,y,t)$, at site (x,y) at time t, has the following forms for Propagating wave, Standing wave and Uniformly oscillating field respectively,

$$h^z(x,y,t) = h_0 cos\{2\pi(ft - \frac{x}{\lambda})\} \tag{8}$$

$$h^z(x,y,t) = h_0 sin(2\pi ft) sin(2\pi\frac{x}{\lambda}) \tag{9}$$

$$h^z(x,y,t) = h_0 cos(2\pi ft) \tag{10}$$

Here h_0 and f represent respectively *the field amplitude and the frequency* for the propagating magnetic wave, standing magnetic wave as well as uniformly oscillating field, whereas λ represents *the wavelength* for both the waves. In this present study I have taken a 100×100 square lattice with periodic boundary conditions for Ising spins which preserve the translational invariance of the system. The magnitude of the spin S is normalised to unity and has $(2S+1)$ number of states. I have taken only four values for S; $(S = 1, \frac{3}{2}, 2, 3)$ to study their general behaviour.

Two distinct dynamical phases namely: *symmetric* phase and *symmetry-broken* phase (see fig.11,14,17.) are observed depending on the values of temperature, strength of the magnetic field and the number of states of the Ising spins for all kinds of magnetic excitations. For a fixed value of S and the amplitude of the magnetic field wave the system undergoes a dynamical phase transition from symmetric (disordered) phase, to symmetry-broken (ordered) phase as the temperature of the system is cooled down below certain critical temperature. Using Monte-Carlo method, the spins of the ferromagnet are parallelly updated at Metropolis rate given by the equation(1). For a fixed field amplitude, the dynamic transition temperature decreases to a limiting value as the number of spin states S increases. Transitions occur also at lower temperature for higher magnetic field amplitudes for a fixed number of spin states. The symmetric phase propagates coherently (see fig.20.) with the propagating magnetic wave whereas in case of standing magnetic wave alternate bands of spins oscillate out of phase (see fig.22.) forming standing waves of spin bands in symmetric phase. For uniformly oscillating field the spins oscillates

coherently with the magnetic field (see fig.25). Below transition temperature the spins orient in some fixed direction (either up or down) and are not affected by the magnetic fields, to yield the maximum for the absolute value of magnetisation (see fig.11,14,17.). This represents the low temperature symmetry broken pinned phase or spin frozen phase. The time averaged magnetisation over a full cycle of the magnetic field oscillation is the dynamic order parameter Q. In pinned phase dynamic order parameter has high value (see fig.12,15,18.) as most of the spins orient in a fixed direction, whereas, in propagating phase or coherently oscillating phase the average magnetisation and hence the dynamic order parameter is nearly equal to zero. The transitions are detected by observing the variations of Q, $\frac{dQ}{dT}$, variance of order parameter $V(= L^2(< Q^2 > - < Q >^2))$ and dynamic specific heat C_v with T (see fig.12,15,18.). Transition temperatures are found from the peaks in the V-T or C_v-T curves and are employed to draw the phase boundaries. It is found that the phase boundary (see fig.13,16,19.) shrinks inwards for high value of spin state S. Also the phase boundaries in the low temperature and high field region are closely spaced as compared to that in the high temperature and low field region. Though the qualitative nature of the nonequilibrium phase transition driven by different magnetic excitations are more or less same in case of multiple state Ising spins the morphological structures (see fig.21,23,24.) of the dynamical spin configurations are different for different kind of magnetic field.

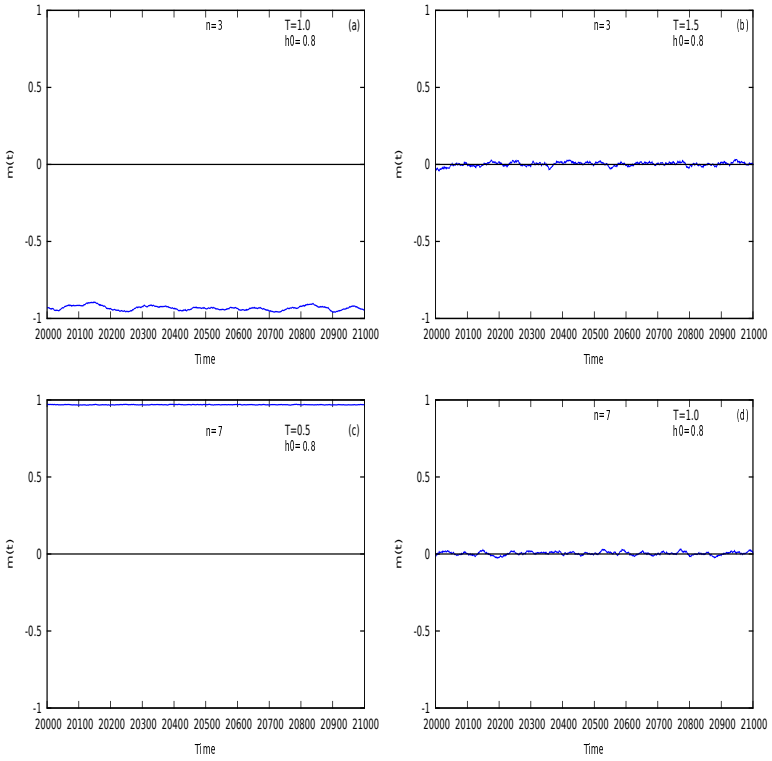

Figure 11: Variation of magnetisation with time at constant temperature (T) and field amplitude (h_0) for propagating wave. Fig.1a & fig.1b. represent *3-state* spin whereas fig.1c. & fig.1d. represent *7-state* spin. Frequency $f = 0.01$.

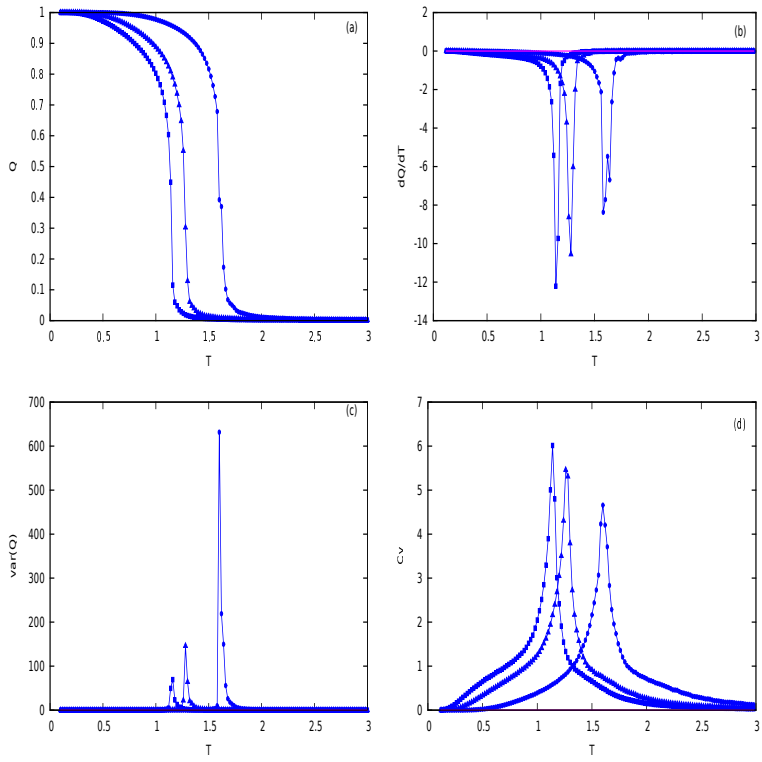

Figure 12: Temperature variation of (a) Q, (b) $\frac{dQ}{dT}$, (c) V, (d) C_v for *3-state* (bullet), *5-state* (uptriangle) and *7-state* (square) spins for constant field amplitude ($h_0 = 0.2$) and frequency ($f = 0.01$) of propagating wave.

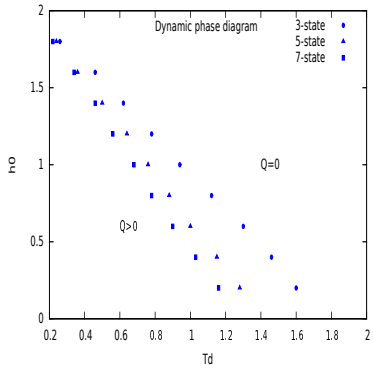

Figure 13: Phase diagram in T_d-h_0 plane for *3-state* (bullet), *5-state* (uptriangle) and *7-state* (square) spins for propagating wave.

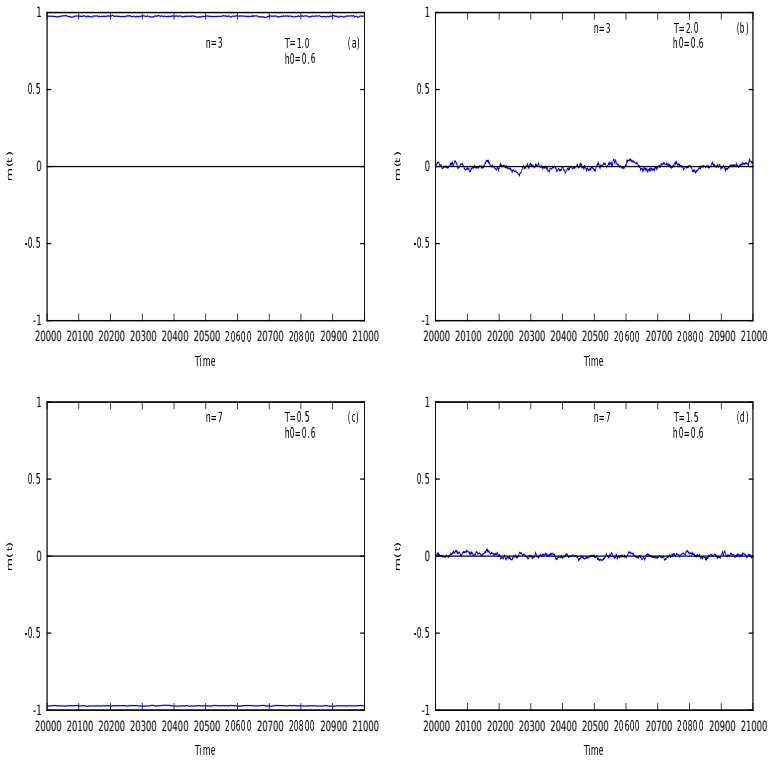

Figure 14: Variation of magnetisation with time at constant temperature (T) and field amplitude (h_0) for standing wave. Fig.4a & fig.4b. represent *3-state* spin whereas fig.4c. & fig.4d. represent *7-state* spin. Frequency $f = 0.01$.

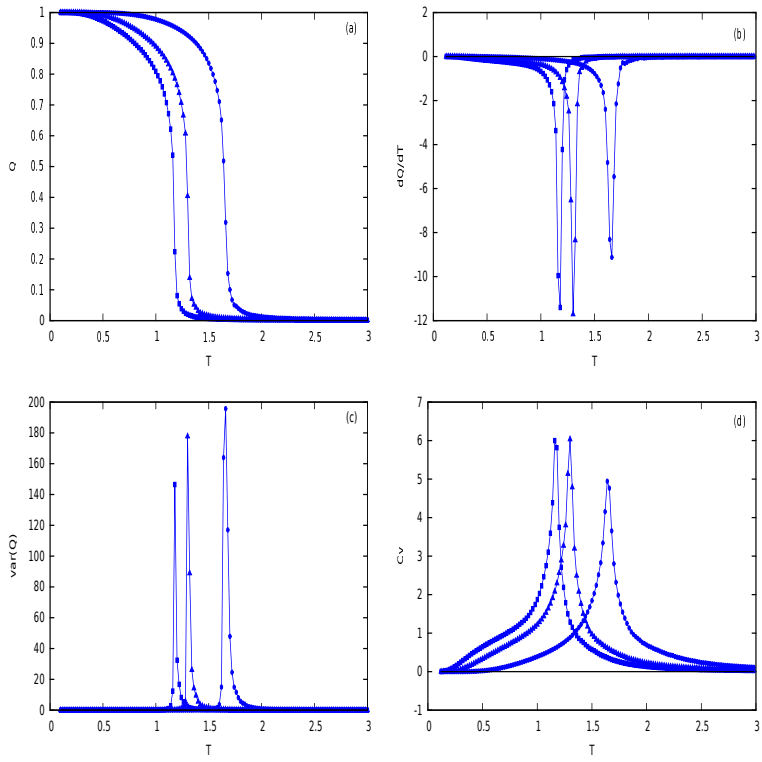

Figure 15: Temperature variation of (a) Q, (b) $\frac{dQ}{dT}$, (c) V, (d) C_v for *3-state* (bullet), *5-state* (uptriangle) and *7-state* (square) spins for constant field amplitude ($h_0 = 0.2$) and frequency ($f = 0.01$) of standing wave.

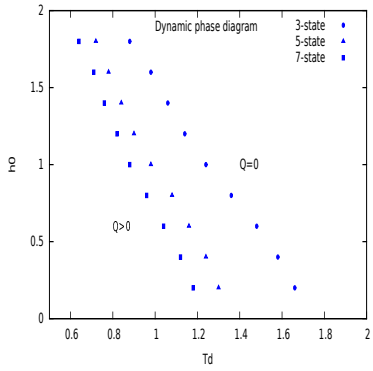

Figure 16: Phase diagram in T_d-h_0 plane for *3-state* (bullet), *5-state* (uptriangle) and *7-state* (square) spins for standing wave.

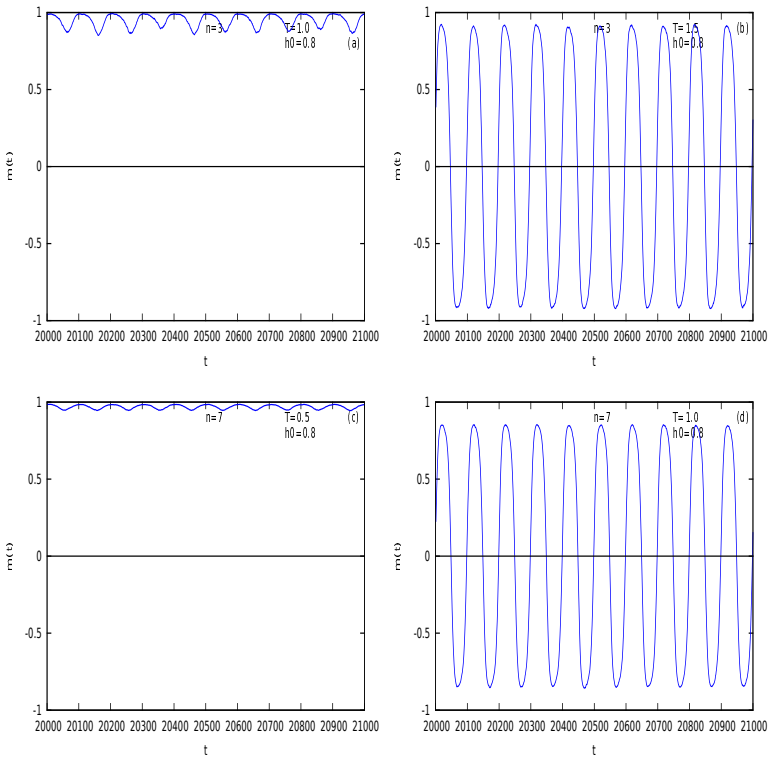

Figure 17: Variation of magnetisation with time at constant temperature (T) and field amplitude (h_0) for uniformly varying field. Fig.7a & fig.7b. represent *3-state* spin whereas fig.7c. & fig.7d. represent *7-state* spin. Frequency $f = 0.01$.

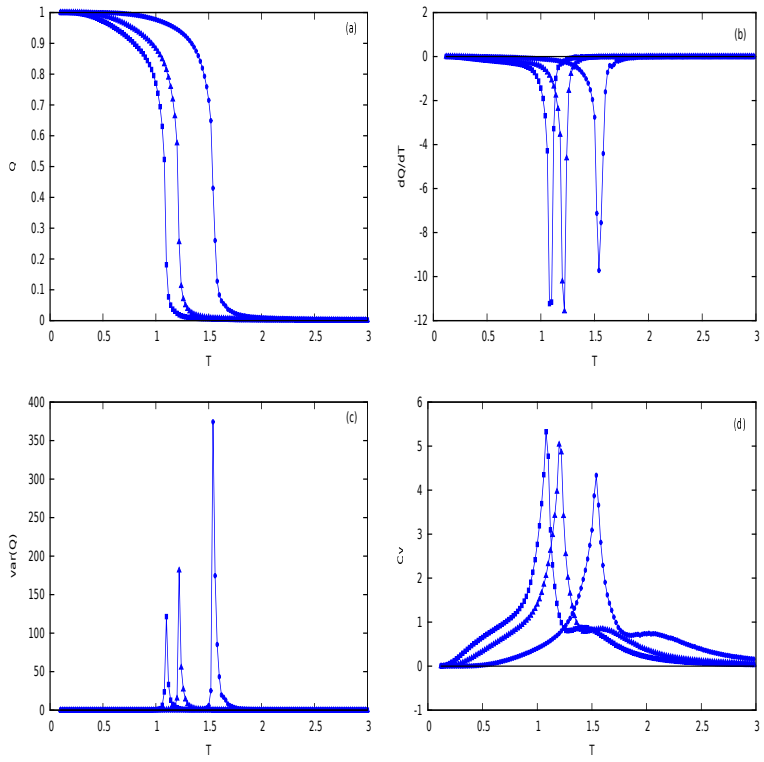

Figure 18: Temperature variation of (a) Q, (b) $\frac{dQ}{dT}$, (c) V, (d) C_v for *3-state* (bullet), *5-state* (uptriangle) and *7-state* (square) spins for constant field amplitude ($h_0 = 0.3$) and frequency ($f = 0.01$) of uniformly varying field.

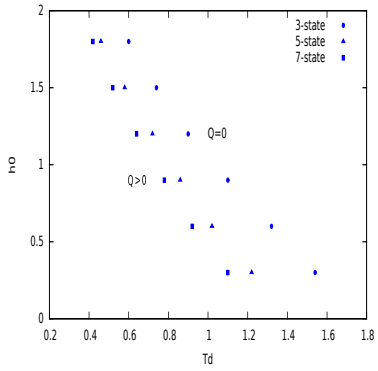

Figure 19: Phase diagram in T_d-h_0 plane for *3-state* (bullet), *5-state* (uptriangle) and *7-state* (square) spins for uniformly varying field.

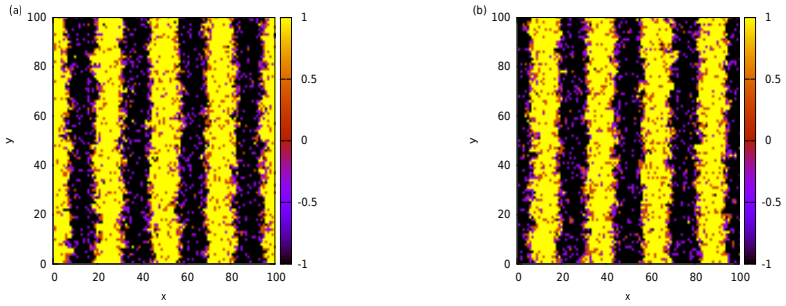

Figure 20: Coherent propagation of spin-wave is shown at two different times for propagating magnetic wave: (a) at $t = 4018$ and (b) at $t = 4067$. Here $n = 4$, $h_0 = 1.0$ & $T = 1.2$.

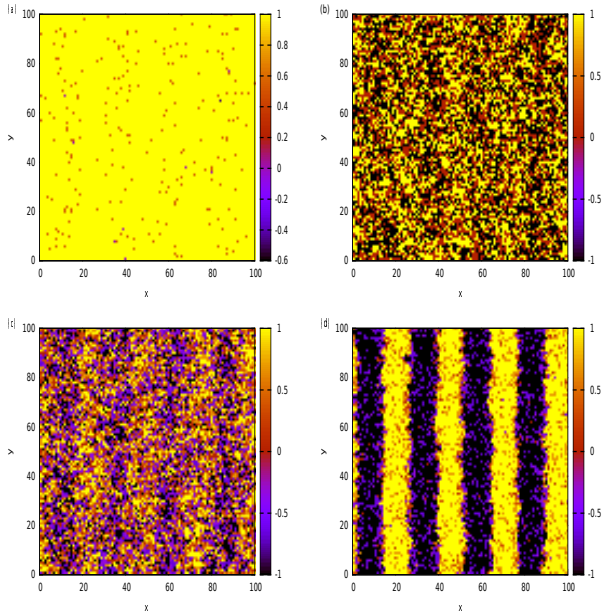

Figure 21: Lattice morphology (value of $S^z(x, y, t)$) at time $t = 4000$ for different values of n, h_0 and T for propagating magnetic wave: (a) $n = 5$, $h_0 = 0.8$ & $T = 0.5$, (b) $n = 3$, $h_0 = 0.2$ & $T = 2.8$, (c) $n = 7$, $h_0 = 0.3$ & $T = 1.8$ and (d) $n = 7$, $h_0 = 1.2$ & $T = 0.8$.

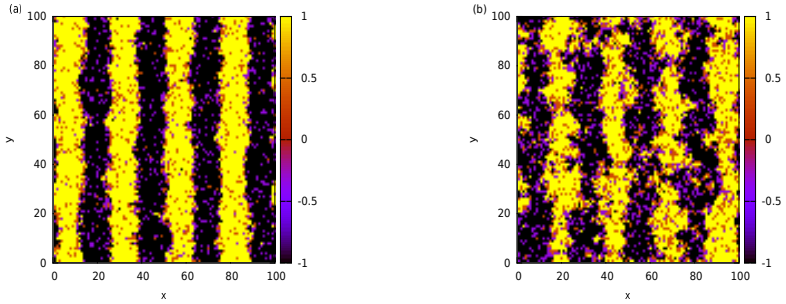

Figure 22: Morphologies of standing wave dynamical modes (non-propagating) for standing magnetic wave are shown at two different times: (a) at $t = 4040$ and (b) at $t = 4067$. Here $n = 4$, $h_0 = 1.2$ & $T = 1.2$.

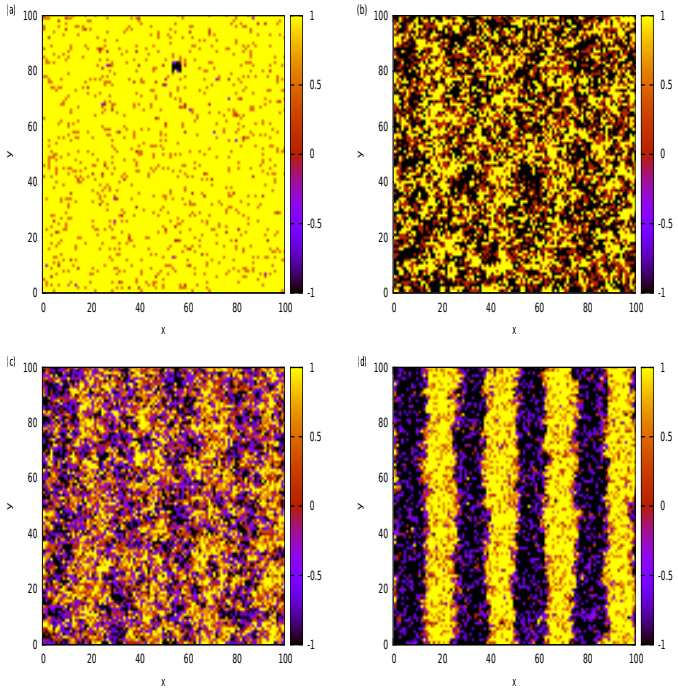

Figure 23: Lattice morphology (value of $S^z(x, y, t)$) at time $t = 4000$ for different values of n, h_0 and T for standing magnetic wave: (a) $n = 5$, $h_0 = 0.8$ & $T = 0.8$, (b) $n = 3$, $h_0 = 0.2$ & $T = 2.2$, (c) $n = 7$, $h_0 = 0.3$ & $T = 1.5$, & (d) $n = 7$, $h_0 = 1.2$ & $T = 1.0$.

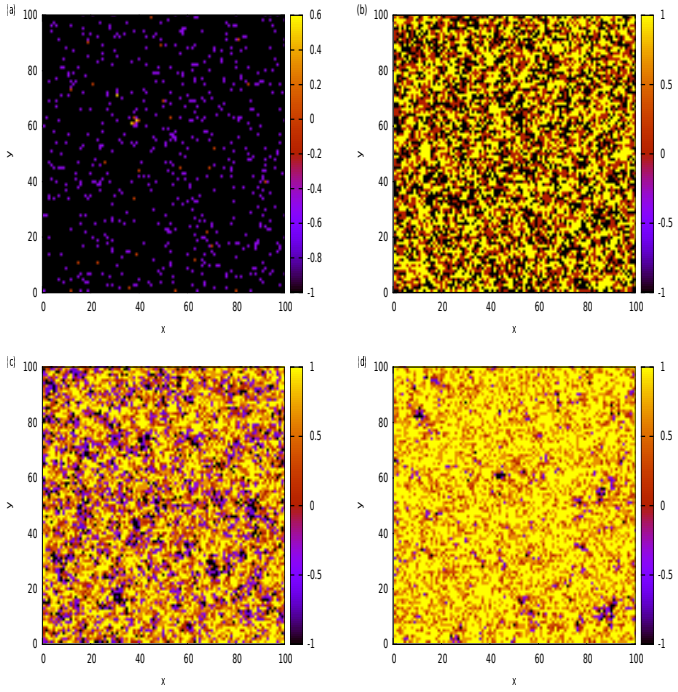

Figure 24: Lattice morphology (value of $S^z(x, y, t)$) for different values of n, h_0 and T for uniformly oscillating magnetic field: (a) $n = 5$, $h_0 = 0.8$ & $T = 0.8$ at time $t = 3950$ and (b) $n = 3$, $h_0 = 0.2$ & $T = 2.9$, (c) $n = 7$, $h_0 = 0.3$ & $T = 1.5$, & (d) $n = 7$, $h_0 = 1.2$ & $T = 1.0$ at time $t = 4000$.

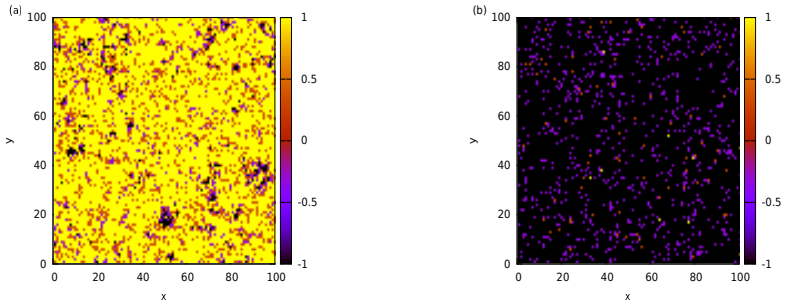

Figure 25: Oscillation of spins shown at different times ($t = 4000$ & $t = 4067$) in symmetric phase for uniformly oscillating magnetic field. Here $n = 4$, $h_0 = 1.2$ & $T = 1.2$.

3.4. Universality class of Ising ferromagnet driven by propagating magnetic wave:

Till now I have studied[40, 41, 42] the dynamic phase transition of different kinds of Ising ferromagnet driven by magnetic waves (mainly propagating and standing magnetic waves) and tried to understand their various dynamical states, dependence of transition temperature on various parameters of magnetic waves and system parameter for a fixed lattice size. However, the detailed finite size analysis, were not yet performed to know the *universality class* of this nonequilibrium phase transition observed in Ising ferromagnet driven by propagating magnetic field wave. This is the key issue of the present study. Recently, similar study of universality class has been done in the BC model and triangular lattice under an oscillating magnetic field[43, 44]. In this work[47], I have investigated the nonequilibrium behaviour, the finite size analysis and the critical aspects of the dynamic phase transition near the dynamic transition temperature of an $L \times L$ square type spin-$\frac{1}{2}$ Ising ferromagnet driven by propagating magnetic wave by Monte Carlo methods. I have taken four different sizes of square lattice ($L = 16,\ 32,\ 64$ *and* 128) for finite size analysis. The ferromagnetic nonequilibrium system transits from a high temperature dynamically symmetric (*propagating*) phase to a low temperature dynamically symmetry-broken (*pinned*) phase as the system is cooled down below the transition temperature (see fig.26a.). This transition temperature is determined precisely by studying the fourth order Binder Cumulant (see fig.26b.) of the dynamic order parameter as a function of temperature for different system sizes (L). From the scaling ansatz of dynamic order parameter ($Q_L \sim L^{-\frac{\beta}{\nu}}$) and the dynamic susceptibility ($\chi_L^Q \sim L^{\frac{\gamma}{\nu}}$), I have estimated the critical exponents $\beta/\nu = 0.146 \pm 0.025$ and $\gamma/\nu = 1.869 \pm 0.135$ (measured from the data read at the critical temperature obtained from Binder cumulant), and $\gamma/\nu = 1.746 \pm 0.017$ (measured from the peak positions of dynamic susceptibility). Our findings suggest that, within the limits of statistical errors obtained in this study, the estimated values of the critical exponents near the dynamic transition temperature are very close to those for the two dimensional equilibrium Ising ferromagnet. *Hence, I may conclude that such a kind of nonequilibrium phase transition in a 2D- Ising ferromagnetic system driven by propagating magnetic wave belongs to the* **same universality class** *of the two-dimensional equilibrium Ising ferro-para transition (where $\beta/\nu = 1/8$ and $\gamma/\nu = 7/4$).*

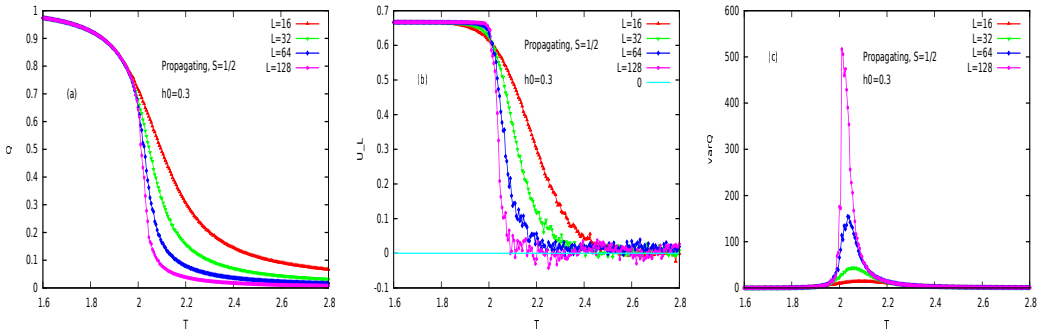

Figure 26: (Color online) Temperature variation of different quantities for different values of linear system size L: (a) Order parameter Q, (b) Binder cumulant U_L , (c) scaled variance of order parameter $varQ$ or susceptibility χ_L^Q.

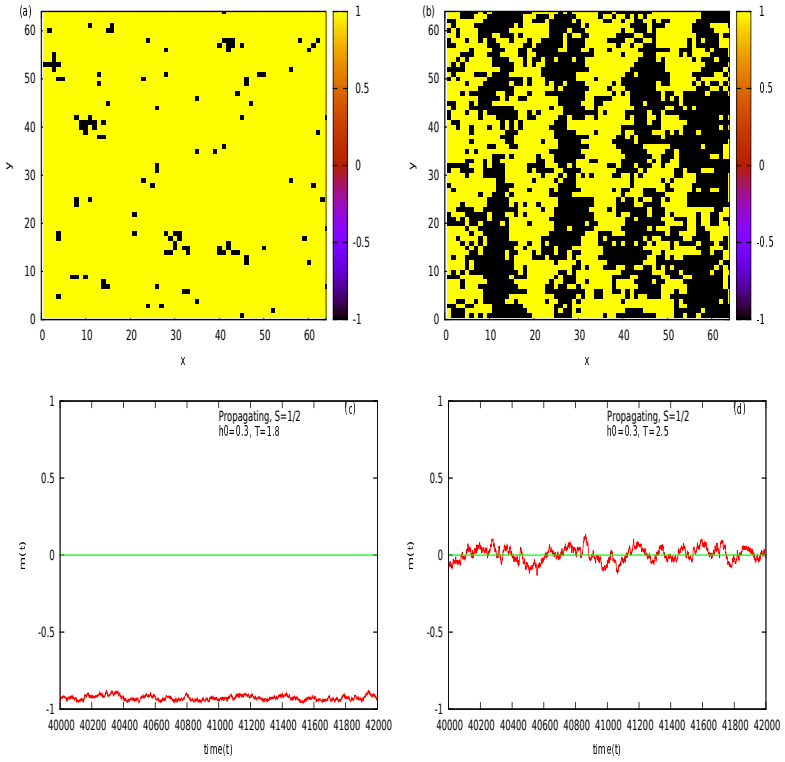

Figure 27: (Color online) Fig.1a. & fig.1b. show the lattice morphology of the pinned and propagating phase respectively at time $t = 39937\ MCSS$ for $L = 64$. Fig.1c. and fig.1d. show the dynamical symmetry breaking (change in the value of average magnetisation per spin from non-zero to nearly zero value). Fig.1a. & fig.1c. are at temperature $T = 1.8$ whereas fig.1b. & fig.1d. are at temperature $T = 2.5$

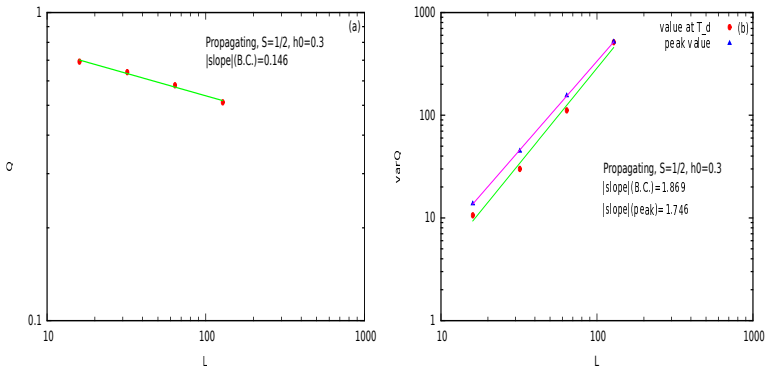

Figure 28: (Color online) Log-log plot of (a) order parameter Q and (b) scaled variance $varQ$ or susceptibility χ_L^Q as a function of linear system size L. In (b) red dots represent the value of susceptibility at T_d whereas blue triangles represent the same at peak positions.

4. Concluding Remarks:

I would say that these present studies are simple statistical approach of achieving various dynamical modes of Ising or Blume-Capel ferromagnet under the intense magnetic field wave of the form like propagating and standing magnetic waves. This may appear that the studies done here are pedagogical in nature. In fact there is a motivation with various experimental background. The site diluted BC model was studied[45] by meanfield renormalization group analysis with good agreement of the experimental phase diagram of Fe-Al alloy. The magnetic alloy of Fe-Al can be modelled by Blume Capel ferromagnet and it would be interesting to see the coherent propagation of spin-bands and the standing spin bands experimentally by time resolved magneto-optic Kerr (TRMOKE) effect. We believe that this study has a significance in the field of spintronics and magnonics[11]. The magnetic behaviours of core-shell magnetic nanoparticles has an importance in the magnetism research as well as in the technology. The magnetic properties have been studied[12] in bimagnetic ($FePt/MFe_2O_4(M = Fe, Co)$) core-shell nanoparticles. The dynamical phase transition has been studied[13] by Monte Carlo simulation in spherical core-shell ($S = \frac{3}{2}$ core and $S = 1$ shell) under time dependent (uniform over space) magnetic field. It would be interesting to study the dynamical behaviours of core-shell magnetic nanoparticles in presence of a magnetic field having spatio-temporal variation appearing as magnetic propagating wave, standing wave etc. An recent study[46] of the magnetic response of the spin-$\frac{1}{2}$ cylindrical nanowire to the propagating magnetic field wave by Monte Carlo simulation method is worth mentioning here.

5A. List of my publications related to Ph.D. thesis:

(i) **Standing magnetic wave on Ising ferromagnet: Nonequilibrium phase transition**: Ajay Halder, Muktish Acharyya[40].
Journal Ref: *J. Magn. Magn. Mater*, **420** (2016) 290;
DOI: 10.1016/j.jmmm.2016.07.062.

(ii) **Blume-Capel ferromagnet driven by propagating and standing magnetic field wave: Dynamical modes and nonequilibrium phase transition**: Muktish Acharyya, Ajay Halder[41].
Journal Ref: *J. Magn. Magn. Mater*, **426** (2017) 53;
DOI: 10.1016/j.jmmm.2016.11.046.

(iii) **Nonequilibrium Phase Transition in Spin-S Ising ferromagnet driven by Propagating and Standing Magnetic Field Wave**: Ajay Halder, Muktish Acharyya[42].
Journal Ref: *Commun. Theor. Phys.*,**68** (2017) 600;
DOI: 10.1088/0253-6102/68/5/600.

(iv) **Universality class of the nonequilibrium phase transition in two dimensional Ising ferromagnet driven by propagating magnetic field wave**: Ajay Halder, Muktish Acharyya[47]
Communicated Ref: arXiv: 1805.02389 [cond-mat.stat-mech].

5B. Conferences participated by me:

(i) Poster presented at International Conference "STATPHYS-VIII", 1-5 December'14, held at the S.N.Bose National Centre for Basic Sciences, Kolkata, India organised by the S.N.Bose National Centre for Basic Sciences, Kolkata, India.

(ii) Poster presented at "ISPC MEETING", 13-15 February'15, held at the Indian Institute of Science, Bangalore, India organised by the International Centre for Theoretical Sciences, Bangalore, India.

(iii) Poster presented at "INTERNATIONAL CONFERENCE ON CONDENSED MATTER PHYSICS", 14-16 November'17, held at the Indian Statistical Institute, Kolkata, India organised by Indian Statistical Institute, Kolkata, India.

(iv) Poster presented at "FRONTIERS OF STATISTICAL PHYSICS", 26-28 February'18, held at the Indian Statistical Institute, Kolkata, India and the Presidency University, Kolkata, India jointly organised by the Indian Statistical Institute, Kolkata, India and the Presidency University, Kolkata, India.

Acknowledgements:

I gratefully acknowledge Prof. Muktish Acharyya from Presidency University, Kolkata for his guidance, active collaboration and very useful discussions. I also like to thank Mrs. Pratima Halder from Presidency University, kolkata for her support and encouragement.

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